Horizontal actions

Modal analysis

The modal analysis is used to determine the vibration modes of the structure, useful to understand the seismic behaviour of the building and to proceed with the linear dynamic analysis.

The modal analysis involves the solution of the generalized eigenvalue problem:

$$[\boldsymbol{K} - \boldsymbol{\Omega}^2 \boldsymbol{M}]\boldsymbol{\Phi} = \boldsymbol{0}$$

where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{M}$ the mass matrix, $\boldsymbol{\Omega}^2$ is the diagonal matrix of the eigenvalues and $\boldsymbol{\Phi}$ is the corresponding matrix of eigenvectors or modal shapes (normalized with respect to the mass matrix); the seismic masses of the diaphragms are calculated with the following combination of vertical loads:

$$\mathrm{G_1 + G_2 + \sum_j \psi_{2j} \cdot Q_{kj}}$$

The eigenvalue, obtained by the solution of the generalized eigenvalue problem, is the square of the circular frequency $\boldsymbol{\omega}$ related to the period, *T*, and to the frequency, *f*, by the following equations:

$$T = \frac{1}{f} \text{ and } f = \frac{\omega}{2\pi}$$

The participating mass ratio for the mode *i-th*, corresponding to an acceleration in the global axis X and Y and to a rotational acceleration around the vertical axis Z, is given by:

$$M_x^i = \frac{m_x^i}{\sum m_{x,j}} \ [\%]$$

$$M_y^i = \frac{m_y^i}{\sum m_{y,j}} \ [\%]$$

$$M_z^i = \frac{m_z^i}{\sum I_{z,j}} \ [\%]$$

where:

$$m_x^i = \frac{\left([\boldsymbol{\Phi}^i]^T \boldsymbol{M}\, \boldsymbol{R}_x\right)^2}{[\boldsymbol{\Phi}^i]^T \boldsymbol{M}\, \boldsymbol{\Phi}^i}$$

$$m_y^i = \frac{\left([\boldsymbol{\Phi}^i]^T \boldsymbol{M}\, \boldsymbol{R}_y\right)^2}{[\boldsymbol{\Phi}^i]^T \boldsymbol{M}\, \boldsymbol{\Phi}^i}$$

$$m_z^i = \frac{\left([\boldsymbol{\Phi}^i]^T \boldsymbol{M}\, \boldsymbol{R}_z\right)^2}{[\boldsymbol{\Phi}^i]^T \boldsymbol{M}\, \boldsymbol{\Phi}^i}$$

and where $\sum m_{x,j}$, $\sum m_{y,j}$ and $\sum I_{z,j}$ are the total masses acting in the axis X, Y and the total rotational inertia about the axis Z of the unrestrained *j-th* degrees of freedom.

Mode	Period [s]	Frequency [Hz]	M_X [%]	Sum M_X [%]	M_Y [%]	Sum M_Y [%]	M_Z [%]	Sum M_Z [%]
Mode 1	0.52	1.91	1.34	1.34	75.13	75.13	4.01	4.01
Mode 2	0.38	2.66	48.90	50.24	7.05	82.18	32.60	36.61
Mode 3	0.29	3.50	45.38	95.61	1.43	83.62	48.00	84.62
Mode 4	0.22	4.51	0.12	95.74	15.97	99.58	4.21	88.82
Mode 5	0.15	6.47	2.93	98.67	0.40	99.99	9.38	98.20
Mode 6	0.10	9.66	1.33	100.00	0.01	100.00	1.80	100.00

Dynamic linear analysis

The dynamic linear analysis consists of:

- the calculation of the seismic effects (the seismic action is represented by the design response spectrum), of each of the vibration modes calculated in the modal analysis;
- combination of these effects.

The seismic effects of the structural model are obtained by the application of the following external loads:

$$\boldsymbol{F_x^i} = \Gamma_x^i \, S_d(T_i) \, \boldsymbol{M} \, \boldsymbol{\Phi^i}$$

e

$$\boldsymbol{F_y^i} = \Gamma_y^i \, S_d(T_i) \, \boldsymbol{M} \, \boldsymbol{\Phi^i}$$

where:

$\boldsymbol{F_x^i}$ and $\boldsymbol{F_y^i}$ are the external loads of the *i-th vibration mode* due to seismic action along X and Y

$S_d(T_i)$ is the spectrum value corresponding to the *i-th* period

$\boldsymbol{\Phi^i}$ is the *i-th modal shape*

$\Gamma_x^i \; and \; \Gamma_y^i$ are the participating modal factor of the *i-th mode* given by:

$$\Gamma_x^i = \frac{[\boldsymbol{\Phi^i}]^T \boldsymbol{M} \, \boldsymbol{R_x}}{[\boldsymbol{\Phi^i}]^T \boldsymbol{M} \, \boldsymbol{\Phi^i}} \quad \text{and} \quad \Gamma_y^i = \frac{[\boldsymbol{\Phi^i}]^T \boldsymbol{M} \, \boldsymbol{R_y}}{[\boldsymbol{\Phi^i}]^T \boldsymbol{M} \, \boldsymbol{\Phi^i}}$$

The effects for a given direction of acceleration (along X or Y) and for each of the vibration modes are combined with the Complete Quadratic Combination technique defined as:

$$E = \left(\sum_j \sum_i \rho_{ij} \cdot E_i \cdot E_j\right)^{1/2}$$

where:

E_j is the effect of the j-th vibration mode;

ρ_{ij} is the correlation coefficient of the *i-th* mode and the *j-th* mode, given by:

$$\rho_{ij} = \frac{8\,\xi^2 \beta_{ij}^{3/2}}{(1+\beta_{ij})\left[\left(1-\beta_{ij}\right)\right]}$$

ξ is the damping ratio in the i-th and j-th modes;

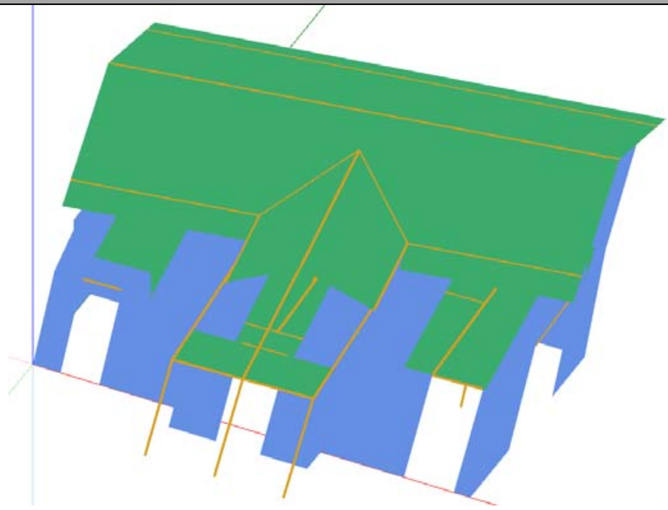
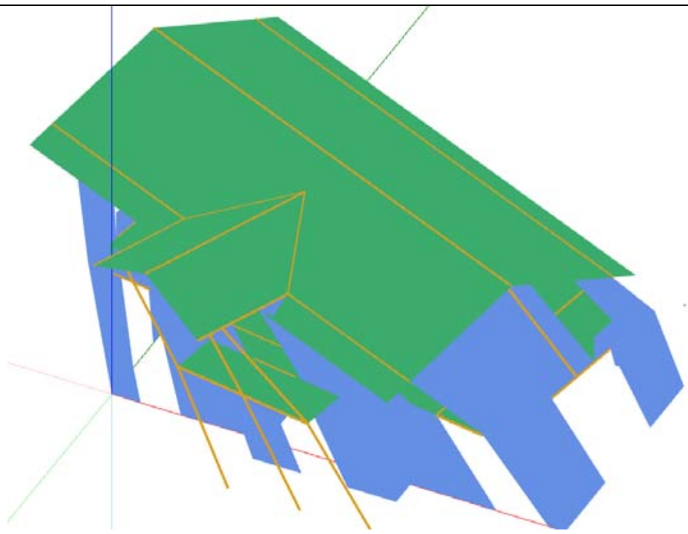
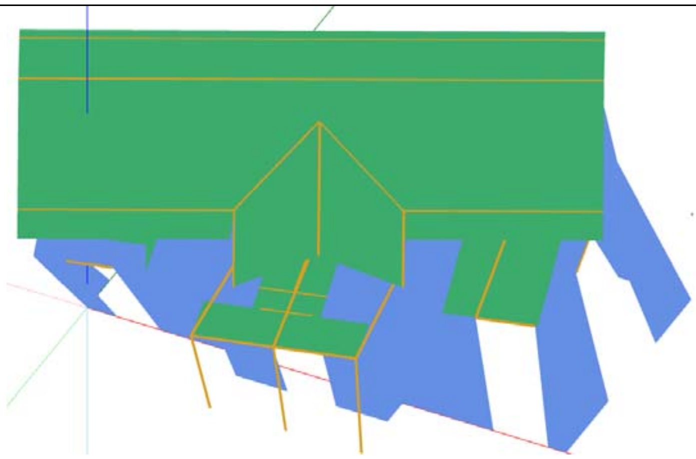
$\beta_{ij} = T_j/T_i$).

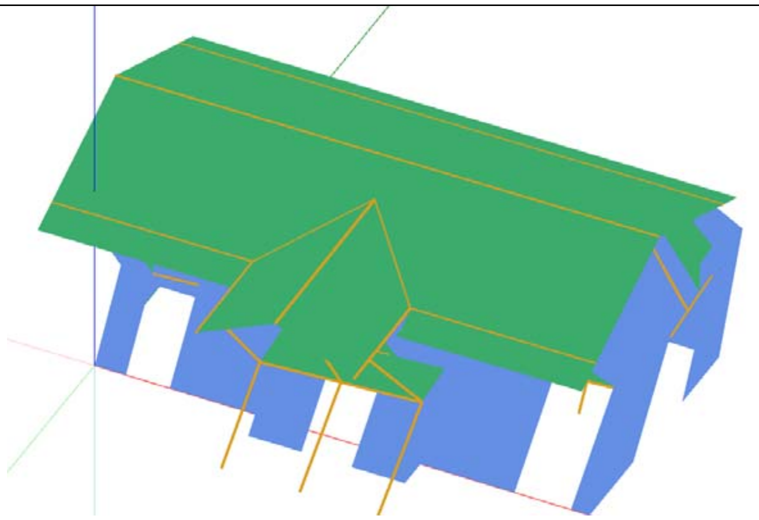
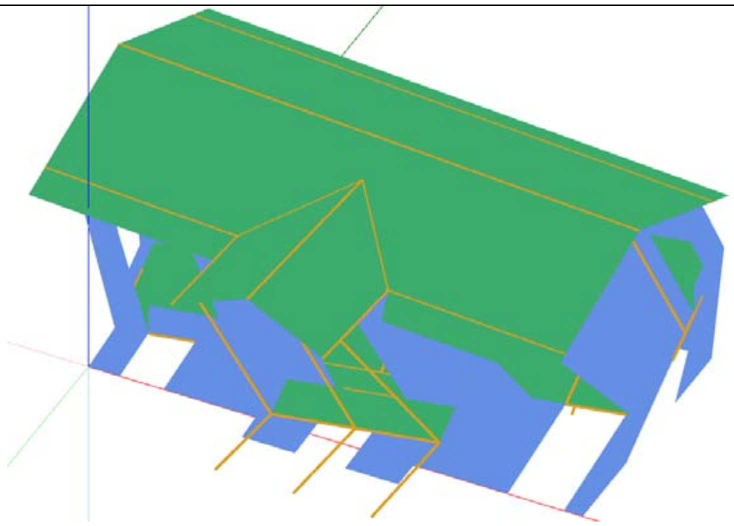
The following table provides the diaphragms properties.

Diaphragm	Height above foundation zi [m]	x_G [m]	y_G [m]	Mass i [kg]
1	3.20	7.71	3.75	68621
2	6.69	7.62	3.62	61793

The following table provides, for each of the vibration modes, the corresponding period and the values of the response spectra.

Mode	Period [s]	SLV spectrum value [g]	SLV spectrum value [g]
Mode 1	0.54	0.10	0.06
Mode 2	0.35	0.12	0.10
Mode 3	0.30	0.12	0.11
Mode 4	0.23	0.12	0.11
Mode 5	0.14	0.12	0.11
Mode 6	0.11	0.11	0.10

Mode	Deformation
Mode 1	
Mode 2	
Mode 3	

Mode 4	
Mode 5	
Mode 6